Puzzle: Why don't the simple formula work?

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Problem: Write a program to find sum of all multiples of 3 and 5 upto input n. If a number is multiple of both 3 and 5 add it only once.

We were given this problem at work. I tried to "solve" it using math. Clearly, we can add all the multiples of 3 and 5 upto n and subtract multiples of 15 once to account for the overcounting. We could derive formula for this as follows:

$$
\begin{align}
f(n) &= (3 + 6 + 9 + \ldots) + (5 + 10 + 15 + \ldots) - (15 + 30 + 45 + \ldots) \tag{1} \\
&= 3(1 + 2 + 3 + \ldots) + 5(1 + 2 + 3 + \ldots) - 15(1 + 2 + 3 + \ldots) \tag{2} \\
&= 3S(n/3) + 5S(n/5) - 15S(n/15) \quad \text{where } S(k) := \frac{1}{2}k(k+1) \tag{3} \\
&= 3 \times \frac{1}{2}\frac{n}{3}\left(\frac{n}{3} + 1\right) + 5 \times \frac{1}{2}\frac{n}{5}\left(\frac{n}{5} + 1\right) - 15 \times \frac{1}{2}\frac{n}{15}\left(\frac{n}{15} + 1\right) \tag{4} \\
&= \frac{n}{2}\left[\frac{n}{3} + 1 + \frac{n}{5} + 1 - \frac{n}{15} - 1\right] \tag{5} \\
&= \frac{n}{2}\left[\frac{n}{3} + \frac{n}{5} - \frac{n}{15} + 1\right] \tag{6}
\end{align}
$$

We write code to do the calculations for us like this:

```
def s(n):
        return (n * (n + 1)) // 2

def ss(n):
        return 3 * s(n//3) + 5 * s(n//5) - 15 * s(n//15)

def sss(n):
        return (n // 2) * ( n // 3 + n // 5 - n // 15 + 1)


tests = [(3, 3), (5,8),(0,0), (10,23+10), (1000, 233168+1000)]

for (n, expected) in tests:
```

```
        print(n, expected, ss(n), sss(n))
        # print(n, expected, ss(n))
        assert ss(n) == expected

# Outputs:
# 3 3 3 2
# 5 8 8 6
# 0 0 0 0
# 10 33 33 30
# 1000 234168 234168 234000
```

Can you figure out why **ss** gives the right answer but **sss** doesn't? Math is straightforward isn't it? Enjoy!